Adaptive Critic Learning Control of Nonlinear Wind Turbine Systems via Integral Event-Triggered Scheme

Shen Yan, Zhou Gu, *Senior Member, IEEE*, and Xiangpeng Xie, *Senior Member, IEEE*

***Abstract*—This brief addresses an integral event-triggered control approach combined with an adaptive critic learning strategy of nonlinear wind turbine systems. To address the optimal control issue, an adaptive critic learning strategy is utilized to approximate the cost function and the solution of the control law. So as to further reduce communication and calculation load, an integral event-triggered scheme is utilized to decrease the releasing rate of control signal. By integrating the triggering law in traditional static event-triggered scheme, the inter-event time can be enlarged, which is proved strictly by a proposition. In addition, the closed-loop system stability and the convergence of the weights of critic neural network are proved with the aid of Lyapunov method. Finally, simulation outcomes are derived to demonstrate the merits of the integral event-triggered adaptive critic learning control approach.**

***Index Terms*—Adaptive critic learning control, integral event-triggered scheme, optimal control, nonlinear wind turbine systems.**

I. INTRODUCTION

WITH the increasing demand for clean and sustainable energy sources, wind energy has emerged as a viable alternative to traditional power generation methods. Wind turbines play a crucial role in converting wind energy into electrical power, which are viewed as a key component of modern renewable energy systems [1]. However, the efficient control of wind turbine systems (WTSs) remains a significant challenge due to their inherent nonlinear and complex dynamics. To cope with this challenge, much effort has been devoted to modeling the nonlinear WTSs as T-S fuzzy systems and some interesting results can be found in [2], [3] and their references. Recently, adaptive critic learning (ACL) technique becomes a popular and effective method to handle the optimal control issues of nonlinear systems, which is also known as adaptive critic design (ACD) [4], [5], [6]. As pointed out by [4], approximate dynamic programming (ADP) and reinforcement learning (RL) are often viewed as the member of ACL family due to the fact that their characteristics in solving optimization problems are almost the same. The main structure of ACL is composed of actor and critic neural networks (NNs) to deal with optimization problems forward in time [7]. A significant advantage of using the actor–critic structure is that "the curse of dimensionality" can be eliminated successfully [8]. The single critic network approximates the cost function (CF) and the optimal controller more conveniently and requires less computation burden than actor-critic neural networks (NNs). Therefore, this brief focuses on the optimal control of nonlinear WTSs by ACL technique.

It is known that the conventional periodic (time-triggered) communication scheme usually requires high computational and computation burden to maintain the given control performance. The beneficial effect of event-triggered scheme (ETS) in mitigating data transmissions and economizing computation and communication resources is widely acknowledged [9], [10], [11]. For ETS, a new control signal is sent to update the controller only when the designed triggering law is violated. Over the past decade, there exist some interesting results to investigate the optimal control issues by combining ACL technique and ETS in [4], [12], [13]. To be specific, an event-driven H_∞ control problem is studied by utilizing ACL method in [4] for nonlinear systems subject asymmetric input constraints. The event-triggered ACL tracking control issue of robotic systems with unknown uncertainties is studied in [12], where a critic NN is adopted to obtain the approximations of the optimal control and the CF. In [13], with the aid of ACL technique, the event-triggered H_∞ stabilization problem of disturbed nonlinear systems is addressed by employing a data driven learning identifier. It is noted that in the above outcomes, most attention is paid to the study on static ETS, which is always required to be negative. In fact, to ensure the system stability, this requirement is not necessary and can be further relaxed. Recently, the investigation of integral ETS (IETS) has gained a lot of attention. In [14], by integrating the triggering condition in static ETS, a new IETS is proposed for the stability of nonlinear systems. the concept. By utilizing the integral of system states to design the triggering condition, the integral event-triggered control strategy is developed in [15], which leads to better data transmission performance. To our knowledge, few effort has yet been made to investigate

Manuscript received 27 November 2023; revised 21 March 2024; accepted 15 April 2024. Date of publication 26 April 2024; date of current version 4 September 2024. This work was supported in part by the National Natural Science Foundation of China under Grant 62103193, Grant 62273183, and Grant 62373196, and in part by the Natural Science Foundation of Jiangsu Province of China under Grant BK20200769 and Grant BK20231288. This brief was recommended by Associate Editor M. Chen. *(Corresponding authors: Zhou Gu; Xiangpeng Xie.)*

Shen Yan is with the College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China (e-mail: yanshenzdh@gmail.com).

Zhou Gu is with the School of Electrical Engineering, Anhui Polytechnic University, Wuhu 241000, China (e-mail: gzh1808@163.com).

Xiangpeng Xie is with the School of Internet of Things, Nanjing University of Posts and Telecommunications, Nanjing 210023, China (e-mail: xiexiangpeng1953@163.com).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2024.3393982.

Digital Object Identifier 10.1109/TCSII.2024.3393982

the integral event-triggered control issue of nonlinear WTSs within the ACL framework. This greatly motivates the current investigation.

Accordingly, this brief deals with the integral event-triggered ACL control problem for nonlinear WTSs. The CF and the implementation of the event-triggered controller are approximated by single critic NN. The main contributions are summarized as below.

1) A novel IETS utilizing the integration of conventional static event-triggering condition and estimated optimal event-triggered control signal is presented to construct the event-triggered ACL controller. Note that the estimated optimal event-triggered control signal are not considered in the existing IETSs in [14], [15], which is infeasible to address the ACL control issue directly in this brief. Compared with [14], [15], our proposed IETS is able to derive the sufficient conditions of guaranteeing the system stability.

2) A new proposition is provided to strictly prove that a larger inter-event time can be generated by our IETS than the static ETS. In contrast to the conventional static ETS, the utilized integration information in our IETS is potential to decrease more redundant triggering events than the static ETS. The advantage of our IETS is also verified by some simulations.

The outline of this brief paper is presented as follows. Section II provides the system modeling and event-triggered ACL control problem formulation. The main results about the stability analysis under the designed IETS and ACL controller are produced in Section III. Simulation results provided in Section IV demonstrate the effectiveness of the developed method. Section V shows the conclusions of this brief.

Notation: In this brief, $A^\top$ stands for the transpose of A. $\nabla(\cdot) = \partial(\cdot)/\partial x$ denotes the gradient operator. $\|\cdot\|$ means the vector norm or the matrix norm. $\lambda_{\min}(\Omega)$ means the minimum eigenvalue of Ω.

II. Preliminaries

A nonlinear WTS studied in [16] is considered as:

$$\begin{cases} \dot{\mathcal{I}}(t) = -\frac{1}{f_1}\mathcal{I}(t) + \frac{f_2}{f_1}\mathcal{V}(t), \\ \dot{\mathcal{W}}(t) = -\frac{f_3}{2f_4}\mathcal{I}(t) + \frac{1}{2f_4}\mathcal{T}(t) - \mathcal{B}\mathcal{W}(t) \\ \mathcal{P}(t) = \mathcal{W}(t) f_3 \mathcal{I}(t), \end{cases} \tag{1}$$

in which $\mathcal{I}(t)$, $\mathcal{V}(t)$ represent the q-axis component of the rotor current and rotor voltage, $\mathcal{W}(t)$ stands for the rotational speed deviation of WT, $\mathcal{P}(t)$ is the output power from WT, $\mathcal{B}$ and f_4 denote the viscous friction and inertial coefficients, respectively, $f_1 = \frac{\mathscr{L}_0}{w_s \mathscr{R}_s}$, $\mathscr{L}_0 = \frac{\mathscr{L}_{rr}+\mathscr{L}_m^2}{\mathscr{L}_{ss}}$, $\mathscr{L}_{rr} = \mathscr{L}_r + \mathscr{L}_m$, $\mathscr{L}_{ss} = \mathscr{L}_s + \mathscr{L}_m$, $f_2 = 1/\mathscr{R}_r$, $f_3 = \mathscr{L}_m/\mathscr{L}_{ss}$ with synchronous speed w_s, stator resistance $\mathscr{R}_s$, leakage inductance $\mathscr{L}_s$, rotor resistance $\mathscr{R}_r$, leakage inductance $\mathscr{L}_r$, magnetizing inductance $\mathscr{L}_m$.

The mechanical power $\mathcal{T}(t)$ is given as

$$\mathcal{T}(t) = \frac{0.5\varrho\pi \mathbf{R}^5 \mathbf{C}_P(\lambda_1, \lambda_2)\mathcal{W}^2(t)}{\lambda_1^3},$$
$$\mathbf{C}_P(\lambda_1, \lambda_2) = \Big(0.44 - 1.67 \times 10^{-2}\lambda_2\Big) \times sin\bigg(\frac{\pi(\lambda_1 - 0.2)}{13 - 0.3\lambda_2} - 1.84 \times 10^{-3}(\lambda_1 - 2)\lambda_2\bigg), \tag{2}$$

in which ϱ is air density, $\mathbf{R}$ is blade radius, λ_1 represents the tip speed ratio and λ_2 denotes the pitch angle.

By choosing $\mathcal{I}(t)$ and $\mathcal{W}(t)$ as the state vector, then a nonlinear WTS can be written as

$$\dot{x}(t) = h(x(t)) + w(x(t))\mu(t), \quad x(0) = x_0, \tag{3}$$

where $x(t) = \begin{bmatrix} \mathcal{I}(t) \\ \mathcal{W}(t) \end{bmatrix} \in \mathbb{R}^2$ is the system state, $\mu(t) = \mathcal{V}(t) \in \mathbb{R}^1$ is the control input, $h(x(t)) = \begin{bmatrix} -\frac{1}{f_1} & 0 \\ -\frac{f_3}{2f_4} & \frac{\mathcal{T}(t)}{2f_4} - \mathcal{B} \end{bmatrix} x(t)$ is the system dynamics and $w(x(t)) = \begin{bmatrix} \frac{f_2}{f_1} \\ 0 \end{bmatrix}$ means the control input matrix. $h(x(t)) + w(x(t))\mu(t)$ is supposed to be Lipschitz continuous, where $h(\cdot)$ is the drift dynamics with respect to $h(0) = 0$.

For system (3), the infinite horizon CF is defined as

$$\mathbb{C}(x) = \int_t^\infty \phi(x, \mu, \theta)d\theta, \tag{4}$$

where $\phi(x, \mu, t) = x^\top(t)\mathsf{M}x(t) + \mu^\top(t)\mathsf{W}\mu(t)$ is the basic utility function with $\mathsf{M} = \mathsf{M}^\top \in \mathbb{R}^{2\times 2}$ and $\mathsf{W} \in \mathbb{R}^{1\times 1}$.

In case the function $\mathbb{C}(x(t), \mu)$ is continuously differentiable, one can obtain the next equation:

$$0 = \phi(x, \mu, t) + \nabla\mathbb{C}(x)^\top\big(h(x(t)) + w(x(t))\mu(t)\big), \tag{5}$$

where $\mathbb{C}(0) = 0$, $\partial\mathbb{C}(x, \mu, t)/\partial x$ is abbreviated as $\nabla\mathbb{C}(x)$.

The definition of Hamilton function for the system (3) is

$$H(x, \mu, \nabla\mathbb{C}(x)) = \phi(x, \mu, t) + \nabla\mathbb{C}(x)^\top(h(x) + w(x)\mu). \tag{6}$$

The definition of the optimal CF and corresponding Hamilton-Jacobi-Bellman (HJB) equation are given as

$$\mathbb{C}^*(x) = \min \int_t^\infty \phi(x, \mu, \theta)d\theta, \tag{7}$$
$$0 = \min H\big(x, \mu(x), \nabla\mathbb{C}^*(x)\big). \tag{8}$$

Then the optimal μ^* is solved as

$$\begin{aligned} \mu^*(x) &= \arg\min H\big(x, \mu^*(x), \nabla\mathbb{C}^*(x)\big) \\ &= -0.5\mathsf{W}^{-1}w^\top(x)\nabla\mathbb{C}^*(x). \end{aligned} \tag{9}$$

Therefore, based on $\phi(x, \mu, t) = x^\top(t)\mathsf{M}x(t) + \mu^\top(t)\mathsf{W}\mu(t)$, the HJB equation (8) becomes

$$\begin{aligned} 0 = {} & x^\top\mathsf{M}x + \big(\nabla\mathbb{C}^*(x)\big)^\top h(x) \\ & - \frac{1}{4}\big(\nabla\mathbb{C}^*(x)\big)^\top w(x)\mathsf{W}^{-1}w^\top(x)\nabla\mathbb{C}^*(x) \end{aligned} \tag{10}$$

with respect to $\mathbb{C}^*(0) = 0$.

III. Main Results

This section presents an integral event-triggered critical learning control that produces an optimal solution to HJB equation.

A. Adaptive Critic Learning Control

The following critic NN is created to derive the approximation of $\mathbb{C}^*(x)$:

$$\mathbb{C}^*(x) = \rho^\top\delta(x) + \epsilon(x), \tag{11}$$

where $\rho \in \mathbb{R}^l$ denotes optimal weight vector, $\delta(x) : \mathbb{R}^n \to \mathbb{R}^l$ means activate function, $\epsilon(x)$ stands for reconstruction error.

We calculate its gradient $\nabla\mathbb{C}^*(x)$ as

$$\nabla\mathbb{C}^*(x) = \nabla\delta^\top(x)\rho + \nabla\epsilon(x) \tag{12}$$

with $\nabla\delta(x) = \partial\delta(x)/\partial x$, $\nabla\epsilon(x) = \partial\epsilon(x)/\partial x$.

Substituting (11) to (9) yields the optimal control policy:

$$\mu^*(x) = -\frac{1}{2}\mathsf{W}^{-1}w^\top(x)\Big(\nabla\delta^\top(x)\rho + \nabla\epsilon(x)\Big). \tag{13}$$

Since $\epsilon(x)$ is unknown in practical learning procedure, the estimation of $\mu(x)$ is approximated by

$$\hat{\mu}(x) = -\frac{1}{2}\mathsf{W}^{-1}w^\top(x)\nabla\delta^\top(x)\hat{\rho}, \tag{14}$$

where $\hat{\rho}$ represents the estimation of ρ. Based on (4), we can rewrite the Bellman's equation as

$$\mathbb{C}(x(t-T)) = \int_{t-T}^{t}\phi(x,\mu,\theta)d\theta + \mathbb{C}(x) \tag{15}$$

for the integration interval $t \in [t-T,\ t]$.

Thence, the Bellman's reconstruction error is computed as

$$\int_{t-T}^{t}\Big(x^\top\mathsf{M}x - \mu^\top\mathsf{W}\mu\Big)d\theta + \rho^\top\Delta\delta(t) = e_b, \tag{16}$$

where $\Delta\delta(t) = \delta(x(t)) - \delta(x(t-T))$.

For unknown $\epsilon(x)$, the actual approximation of e_b is obtained as

$$\int_{t-T}^{t}\Big(x^\top\mathsf{M}x - \mu^\top\mathsf{W}\mu\Big)d\theta + \hat{\rho}^\top\Delta\delta(t) = e_r. \tag{17}$$

To get the minimum value of e_r in (17), the critic objective function $\frac{1}{2}e_r^\top e_r$ is chosen. Next, with the utilization of gradient descent approach, $\frac{1}{2}e_r^\top e_r$ can be minimized based on the gradient item:

$$\frac{\partial\frac{1}{2}e_r^\top e_r}{\partial\hat{\rho}} = \frac{\partial e_r}{\partial\hat{\rho}}e_r^\top. \tag{18}$$

Thus, the training law of the critic weight is presented as

$$\dot{\hat{\rho}} = -\alpha\frac{\Delta\delta(t)}{\big(\Delta\delta^\top(t)\Delta\delta(t)+1\big)^2}e_r, \tag{19}$$

where $\alpha > 0$ denotes the learning rate, and $(\Delta\delta^\top(t)\Delta\delta(t)+1)^2$ is adopted to normalization.

B. Integral Event-Triggered Adaptive Critic Learning Control

Under ETS case, a new control signal $\mu(t)$ is generated only when a new triggered data is received by controller. Usually, a zero-order-hold (ZOH) is used to maintain $\mu(t) = \mu(t_k)$ for $t \in [t_k,\ t_{k+1})$, where t_k and t_{k+1} are the current and next triggering instants, respectively. The state error is defined as

$$e_k(t) = x(t_k) - x(t), \tag{20}$$

where $x(t_k)$ is simplized as x_k in the following derivations.

Next, $\hat{\mu}(x)$ under ETS can be written as

$$\hat{\mu}(x_k) = -\frac{1}{2}\mathsf{W}^{-1}w^\top(x_k)\nabla\delta^\top(x_k)\hat{\rho}. \tag{21}$$

Before further proceeding, two widely used assumptions are given as below.

Assumption 1 [17]: There exists a positive scalar ζ such that the Bellman's error satisfies $\|e_b\| \le \zeta$.

Assumption 2 [12], [13]: $\mu(t)$ is assumed to be Lipschitz continuous and satisfy $\|\mu(x) - \mu(x_k)\| \le \mathsf{K}\|e_k(t)\|$ for a scalar $\mathsf{K} > 0$.

In order to further save precious network resource, the following IETS is designed to generate the next triggering instant:

$$t_{k+1} = \arg\min_t\{t > t_k | \eta(t) \ge 0\}, \tag{22}$$

where $\dot{\eta}(t) = \mathsf{K}^2\|r\|^2\|e_k(t)\|^2 - \chi x^\top\mathsf{M}x - \|r\|^2\|\hat{\mu}(x_k)\|^2$, $\chi \in (0,\ 1)$ and $rr^\top = \mathsf{W}$.

Remark 1: By replacing the integral term $\eta(t)$ in our IETS (22) with $\dot{\eta}(t)$, the scheme reduces to the normal ETS as:

$$t_{k+1} = \arg\min_t\{t > t_k \in \mathbb{R}|\dot{\eta}(t) \ge 0\}, \tag{23}$$

which can be viewed as the existing static ETS.

Theorem 1: For the training policy of critic weight (19) and the event-triggered control (21), the system (3) is with asymptotic stability and the estimation error of critic weight $\tilde{\rho} = \rho - \hat{\rho}$ is uniformly ultimately bounded (UUB) if the condition $\eta(t) \le 0$ holds and the next condition

$$\|\tilde{\rho}\| > \sqrt{\zeta^2/\lambda_{\min}(\Omega)} \tag{24}$$

with $\Omega = \Delta\Psi\Delta\Psi^\top$ holds.

Proof: The Lyapunov function is chosen as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{25}$$

where

$$V_1(t) = \int_{t-T}^{t}\mathbb{C}^*(x)d\theta, \qquad V_2(t) = \int_{t-T}^{t}\mathbb{C}^*(x_k)d\theta,$$
$$V_3(t) = \frac{1}{2}\tilde{\rho}^\top\tilde{\rho}, \qquad V_4(t) = -\int_{t-T}^{t}\eta(\theta)d\theta.$$

According to the triggering condition (22), one obtains the condition $\eta(t) \le 0$, which ensures $V_4(t)$ is non-negative and further guarantees the positiveness of $V(t)$.

The following two cases are provided to prove the theorem for $t \in (t_k, t_{k+1})$ and $t = t_{k+1}$, respectively.

Case A: $t \in (t_k,\ t_{k+1})$. From $\dot{V}_2(t) = 0$ and computing the derivations of $\dot{V}_1(t)$, $\dot{V}_3(t)$ and $\dot{V}_4(t)$, we derive

$$\begin{aligned}\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_3(t) + \dot{V}_4(t)\\ &= \int_{t-T}^{t}\dot{\mathbb{C}}^*(x)d\theta + \tilde{\rho}^\top\dot{\tilde{\rho}} - \int_{t-T}^{t}\dot{\eta}(\theta)d\theta.\end{aligned} \tag{26}$$

In terms of (9), it yields

$$\nabla\mathbb{C}^*(x)w = -2\mu^{*\top}(x)\mathsf{W}, \tag{27}$$

$$\nabla\mathbb{C}^*(x)h = \frac{1}{4}\nabla\mathbb{C}^{*\top}(x)w\mathsf{W}^{-1}w^\top\nabla\mathbb{C}^*(x) - x^\top\mathsf{M}x. \tag{28}$$

Then, one can get

$$\begin{aligned}\dot{\mathbb{C}}^*(x) &= \nabla\mathbb{C}^*(x)\big(h + w\hat{\mu}(x_k)\big)\\ &= \frac{1}{4}\nabla\mathbb{C}^{*\top}(x)w\mathsf{W}^{-1}w^\top\nabla\mathbb{C}^*(x) - x^\top\mathsf{M}x - 2\mu^{*\top}(x)\mathsf{W}\hat{\mu}(x_k)\\ &= \mu^{*\top}(x)\mathsf{W}\mu^*(x) - x^\top\mathsf{M}x - 2\mu^{*\top}(x)\mathsf{W}\hat{\mu}(x_k)\\ &= -\chi x^\top\mathsf{M}x - (1-\chi)x^\top\mathsf{M}x - \|r^\top\hat{\mu}(x_k)\|^2\\ &\quad + \Big\|r^\top\big(\mu^*(x) - \hat{\mu}(x_k)\big)\Big\|^2.\end{aligned} \tag{29}$$

According to Assumption 2, it leads to

$$\begin{aligned}\dot{V}_1(t) \le \int_{t-T}^{t}\Big(&-\chi x^\top\mathsf{M}x + \mathsf{K}^2\|r\|^2\|e_k(t)\|^2\\ &- \|r\|^2\big\|\hat{\mu}(x_k)\big\|^2\Big)d\theta - \int_{t-T}^{t}(1-\chi)x^\top\mathsf{M}xd\theta.\end{aligned} \tag{30}$$

In terms of the weight training law (19), one has

$$\dot{\tilde{\rho}} = -\alpha\Delta\Psi\Delta\Psi^\top\tilde{\rho} + \alpha\Delta\Psi\frac{e_b}{\Delta\delta^\top(t)\Delta\delta(t)+1}, \quad (31)$$

where $\Delta\Psi = \frac{\Delta\delta^\top(t)}{\Delta\delta^\top(t)\Delta\delta(t)+1}$.

From (31), it results in

$$\begin{aligned}\dot{V}_3(t) &= -\alpha\tilde{\rho}^\top\Delta\Psi\Delta\Psi^\top\tilde{\rho} + \alpha\tilde{\rho}^\top\frac{\Delta\Psi}{\Delta\delta^\top(t)\Delta\delta(t)+1}e_b \\ &\le -\alpha\tilde{\rho}^\top\Delta\Psi\Delta\Psi^\top\tilde{\rho} + \alpha\tilde{\rho}^\top\Delta\Psi e_b \\ &\le -\frac{1}{2}\alpha\lambda_{\min}\left(\Delta\Psi\Delta\Psi^\top\right)\|\tilde{\rho}\|^2 + \frac{1}{2}\alpha\zeta^2, \quad (32)\end{aligned}$$

which means $\tilde{\rho}$ is UUB.

Then, based on $\dot{\eta}(t) = \mathsf{K}^2\|r\|^2\|e_k(t)\|^2 - \chi x^\top\mathsf{M}x - \|r\|^2\|\hat{\mu}(x_k)\|^2$, it yields

$$\begin{aligned}&\dot{V}(t) \le -0.5\alpha\lambda_{\min}(\Omega)\|\tilde{\rho}\|^2 + 0.5\alpha\zeta^2 - \int_{t-T}^{t}(1-\chi)x^\top\mathsf{M}xd\theta \\ &+ \int_{t-T}^{t}\left(-\chi x^\top\mathsf{M}x + \mathsf{K}^2\|r\|^2\|e_k(t)\|^2 - \|r\|^2\left\|\hat{\mu}(x_k)\right\|^2\right)d\theta \\ &- \int_{t-T}^{t}\left(-\chi x^\top\mathsf{M}x + \mathsf{K}^2\|r\|^2\|e_k(t)\|^2 - \|r\|^2\left\|\hat{\mu}(x_k)\right\|^2\right)d\theta. \quad (33)\end{aligned}$$

Once the condition (24) is satisfied and $\chi \in (0,\ 1)$, it gives

$$\dot{V}(t) \le -\int_{t-T}^{t}(1-\chi)x^\top\mathsf{M}xd\theta < 0. \quad (34)$$

Case B: $t = t_{k+1}$. Based on Case A, it gives $\dot{V}(t) < 0$ for $t \in (t_k,\ t_{k+1})$.

Then, we have

$$\begin{aligned}&\nabla V_1(t) = \mathbb{C}^*(x_{k+1}) - \mathbb{C}^*\left(x_{k+1}^-\right) \le 0, \\ &\nabla V_2(t) = \mathbb{C}^*(x_{k+1}) - \mathbb{C}^*(x_k), \\ &\nabla V_3(t) = \frac{1}{2}\tilde{\rho}^\top(x_{k+1})\tilde{\rho}(x_{k+1}) - \frac{1}{2}\tilde{\rho}^\top\left(x_{k+1}^-\right)\tilde{\rho}\left(x_{k+1}^-\right) \le 0, \\ &\nabla V_4(t) = \eta(t_{k+1}) - \eta\left(t_{k+1}^-\right) = 0. \quad (35)\end{aligned}$$

In terms of the fact in [18], one can get $\nabla V_2(t) = \mathbb{C}^*(x_{k+1}) - \mathbb{C}^*(x_k) \le -\mathcal{K}(\|x_{k+1} - x_k\|)$, where $\mathcal{K}(\cdot)$ means a class-$\mathcal{K}$ function. It further leads to $\nabla V(t) \le 0$.

By combining Case A and Case B, it ensures the system (3) is asymptotically stable and $\tilde{\rho}$ is UUB. The proof of Theorem 1 ends. ■

Remark 2: By using the integral term $\eta(t)$, the condition $\dot{\eta}(t) = \mathsf{K}^2\|r\|^2\|e_k(t)\|^2 - \chi x^\top\mathsf{M}x - \|r\|^2\|\hat{\mu}(x_k)\|^2$ in static ETS (23) is not required to be less than zero strictly. Namely, with the introduction of $\eta(t)$, our IETS is potential to further relax the static ETS. Then, a larger inter-event time generated by our IETS than the static ETS is proved strictly in the following proposition.

To illustrate the effectiveness of our IETS for reducing the triggering times and providing a larger inter-event time, Proposition 1 is presented as below to prove it.

Proposition 1: Let $t_k = t_k^s = t_k^i$, t_{k+1}^s be given by static ETS (23) and t_{k+1}^i be given by IETS (22), then $t_{k+1}^s \le t_{k+1}^i$.

Proof: If we assume that $t_{k+1}^s > t_{k+1}^i$ and substitute t_{k+1}^i into (23), it yields

$$\dot{\eta}\left(t_{k+1}^i\right) < 0. \quad (36)$$

Integrating (36) over $t \in [t_k,\ t_{k+1}^i]$, one can get

$$\eta\left(t_{k+1}^i\right) - \eta(t_k) = \eta\left(t_{k+1}^i\right) < 0, \quad (37)$$

where $\eta(t_k) = 0$ because the IETS condition (22) is triggered at the time instant t_k.

From the IETS (22), it leads to

$$\eta\left(t_{k+1}^i\right) \ge 0, \quad (38)$$

which contracts to (37).

Hence, $t_{k+1}^s \le t_{k+1}^i$ is proved. ■

TABLE I
SYSTEM PARAMETERS

Symbol	Value	Symbol	Value	Symbol	Value
$\varrho(Kg/m^3)$	1.225	$\mathscr{L}_m(pu)$	52	$\mathcal{B}(pu)$	50
$\mathbf{R}(m)$	5	$\mathscr{L}_s(pu)$	0.07397	$\mathscr{R}_s(pu)$	7.9
$\lambda_2(°)$	0	$\mathscr{L}_r(pu)$	0.002	$\mathscr{R}_r(pu)$	2
$\lambda_1(pu)$	8.1	$w_s(m/s)$	1	$f_4(s)$	0.1

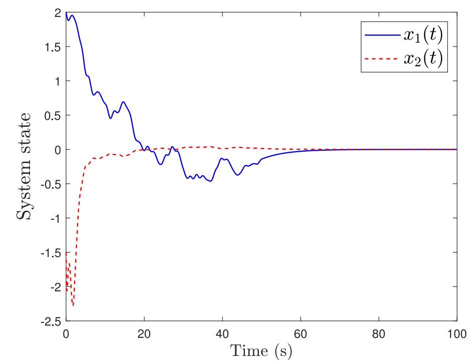

Fig. 1. The system state $x(t)$.

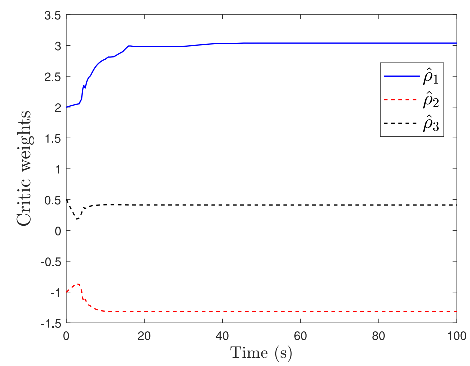

Fig. 2. Estimated critic weights evolution under our IETS.

IV. SIMULATION RESULTS

In the simulation, the WPS parameters are considered in the following table.

Then, we select $x(0) = \begin{bmatrix}2 & -1.5\end{bmatrix}^\top$, and adaptive learning rate $\alpha = 10$ and $\delta(x) = \begin{bmatrix}x_1^2 & x_1x_2 & x_2^2\end{bmatrix}$, $\mathsf{M} = I_2$, $\mathsf{W} = 1$.

Moreover, a probing noise $\mathfrak{N}(t) = 2.5e^{-0.01t}(sin^2(t)cos(t) + sin^2(2t)cos(0.1t) + sin^2(-1.2t)cos(0.5t) + sin^5(t))$ is added to the control input for $t \le 50s$ to improve the training performance of NN weights.

Based on the above parameters, the curves of system state $x(t)$ and under our IETS are drawn in Fig. 1. Fig. 2 shows the evolution of critic weights obtained under our IETS. After a period of tuning, it is observed that the stability of

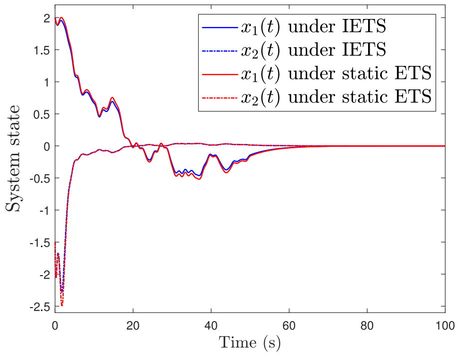

Fig. 3. The system state $x(t)$ under our IETS and static ETS.

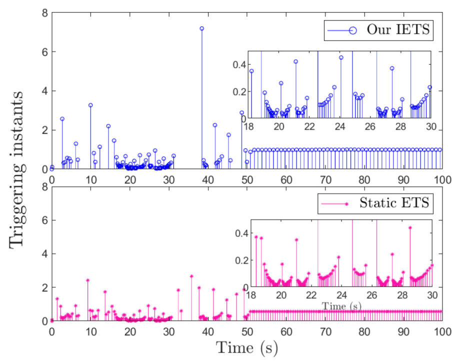

Fig. 4. Triggering instants under our IETS and static ETS.

nonlinear WPS is guaranteed well and the weights of critic NN converges to $\begin{bmatrix}3.0385 & -1.3139 & 0.412\end{bmatrix}$ asymptotically. Moreover, the comparison results of state responses and triggering instants generated by static ETS and our proposed IETS are obtained in Fig. 3, Fig. 4. The numbers of transmitted data under the static ETS (23) and our IETS (22) are 346 and 272, respectively.

In terms of Fig. 3, it is observed that the trajectories of system state $x(t)$ obtained by the two ETSs are very similar. From Fig. 4, the triggering times generated by our IETS (22) is dramatically saved 21.4% compared to the existing static ETS (23). These comparison results demonstrate the superiority of IETS for enlarging the inter-event time.

V. Conclusion

This brief presents a novel control strategy for nonlinear WTSs by combining IETS and ACL. First, an ACL technique is introduced to approximate the CF and the optimal control policy. Second, an IETS in view of the integration of the triggering law in static ETS is proposed to lower the data releasing rate and computation load. A novel proposition is presented to prove that a larger inter-event time is generated by our IETS than the traditional static ETS. Then, the Lyapunov theory is adopted to analyze the stability of the closed-loop system and the convergence of the CNN weights. Finally, the proposed approach is demonstrated to be effective through some simulation results. In the future, it is possible to utilize more information like proportional and derivative signals to design more effective ETSs to further enlarge the inter-event time interval.

References

[1] Y. Zhang, X. Liu, and B. Qu, "Distributed model predictive load frequency control of multi-area power system with DFIGs," *IEEE/CAA J. Automatica Sinica*, vol. 4, no. 1, pp. 125–135, Jan. 2017.

[2] R. Datta and Y. H. Joo, "Fuzzy memory sampled-data controller design for PMSG-based WECS with stochastic packet dropouts," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 12, pp. 4421–4434, Dec. 2023.

[3] S. Yan, Z. Gu, J. H. Park, and X. Xie, "Adaptive memory-event-triggered static output control of TS fuzzy wind turbine systems," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 9, pp. 3894–3904, Sep. 2022.

[4] X. Yang, H. He. "Event-driven H_∞-constrained control using adaptive critic learning," *IEEE Trans. Cybern.*, vol. 51, no. 10, pp. 4860–4872, Oct. 2021.

[5] M. Ha, D. Wang, and D. Liu, "Discounted iterative adaptive critic designs with novel stability analysis for tracking control," *IEEE/CAA J. Automatica Sinica*, vol. 9, no. 7, pp. 1262–1272, Jul. 2022.

[6] Z. Peng et al., "Optimal H_∞ tracking control of nonlinear systems with zero-equilibrium-free via novel adaptive critic designs," *Neural Netw.*, vol. 164, pp. 105–114, Jul. 2023.

[7] D. Vrabie and F. Lewis, "Neural network approach to continuous-time direct adaptive optimal control for partially unknown nonlinear systems," *Neural Netw.*, vol. 22, no. 3, pp. 237–246, 2009.

[8] W. B. Powell, *Approximate Dynamic Programming: Solving the Curses of Dimensionality*, 2nd ed. Hoboken, NJ, USA: Wiley, 2007.

[9] X.-M. Zhang et al., "Networked control systems: A survey of trends and techniques," *IEEE/CAA J. Automatica Sinica*, vol. 7, no. 1, pp. 1–17, Jan. 2020.

[10] X. Ge, S. Xiao, Q.-L. Han, X. M. Zhang, and D. Ding, "Dynamic event-triggered scheduling and platooning control co-design for automated vehicles over vehicular Ad-Hoc networks," *IEEE/CAA J. Automatica Sinica*, vol. 9, no. 1, pp. 31–46, Jan. 2022.

[11] F. Yang, X. Xie, Q. Sun, and D. Yue, "FDI attack estimation and event-triggered resilient control of DC microgrids under hybrid attacks," *IEEE Trans. Smart Grid*, early access, Feb. 21, 2024, doi: 10.1109/TSG.2024.3368076.

[12] Z. Peng, W. Yan, R. Huang, H. Cheng, K. Shi, and B. K. Ghosh, "Event-triggered learning robust tracking control of robotic systems with unknown uncertainties," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 70, no. 7, pp. 2540–2544, Jul. 2023.

[13] D. Wang, C. Mu, D. Liu, and H. Ma, "On mixed data and event driven design for adaptive-critic-based nonlinear H_∞ control," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 993–1005, Apr. 2018.

[14] S. H. Mousavi, M. Ghodrat, and H. J. Marquez, "Integral-based event-triggered control scheme for a general class of non-linear systems," *IET Control Theory Appl.*, vol. 9, no. 13, pp. 1982–1988, 2015.

[15] H. Yu and F. Hao, "Input-to-state stability of integral-based event-triggered control for linear plants," *Automatica*, vol. 85, pp. 248–255, Nov. 2017.

[16] S. Yan, Z. Gu, J. H. Park, and X. Xie, "Sampled memory-event-triggered fuzzy load frequency control for wind power systems subject to outliers and transmission delays," *IEEE Trans. Cybern.*, vol. 53, no. 6, pp. 4043–4053, Jun. 2023.

[17] Z. Zhang, Z. Peng, B. Zhao, R. Huang, J. Hu, and H. Cheng, "Optimal H_∞ control of nonlinear systems via static and dynamic triggering critic learning," in *Proc. 4th Int. Conf. Data-driven Optim. Complex Syst. (DOCS)*, 2022, pp. 1–6.

[18] D. Wang, C. Mu, and D. Liu, "Adaptive critic designs for solving event-based H_∞ control problems," in *Proc. Amer. Control Conf. (ACC)*, 2017, pp. 2435–2440.